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ON THE GENERALIZATION OF RICE FORMULAS AND THEIR APPLICATION FOR THE DETERMINATION OF SPECTRAL DENSITIES

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ON THE GENERALIZATION OF RICE FORMULAS AND THEIR APPLICATION FOR THE DETERMINATION OF SPECTRAL DENSITIES*

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by Gabriel Coupry

SUMMARY

The problem, first of all, is to extend to stationary processes of any distribution functions the formulas proposed by Rice in 1945 for the Gaussian stationary processes. These formulas, which determine the mean number of zeros per second of an uncertain function of t, are at the basis of a new, rapid and precise method to obtain spectral densities. A plate attached will allow the comparison of the spectral densities measured by the classical filtering method with the zero method.

* * *

1. - Considering an uncertain stationary process f (t), M. Kac [1] and S. O. Rice [2] have shown that the probability for the function f (t) to become zero in the interval (t, t + dt) was given by the formula

$$dp' = dt \int_0^{+\infty} \eta p(o, \eta)\, d\eta, \qquad (1)$$

where $p(\xi, \eta)$ is the probability density of the couple of uncertain variables

$$\xi = f(t), \qquad \eta = f'(t),$$

of respective typical gaps σ and σ'.

If we denote by $p(\alpha, \beta)$ the probability density of two reduced variables

$$\alpha = \frac{\xi}{\sigma}, \qquad \beta = \frac{\eta}{\sigma'},$$

* Sur une généralisation des formules de Rice et leur application a la détermination des densités spectrales

the probability density $p(\xi, \eta)$ will be written

$$p(\xi, \eta) = \frac{1}{\sigma\sigma'} \tilde{p}\left(\frac{\xi}{\sigma}, \frac{\eta}{\sigma'}\right).$$

If we substitute this expression into the equations (1), we shall obtain:

$$\frac{dp'}{dt} = \frac{1}{\sigma\sigma'} \int_0^{+\infty} \eta \tilde{p}\left(0, \frac{\eta}{\sigma'}\right) d\eta.$$

By an evident variable substitution we shall derive therefrom the mean number of zeros per second :

$$N = \frac{\sigma'}{\sigma} \int_0^{+\infty} \beta \tilde{p}(0, \beta)\, d\beta,$$

the integral allowing only for the intervention of the reduced probability density.

In the general case when f (t) and F' (t) may be considered as two stochastically independent variables, we may postulate

$$\tilde{p}(\alpha, \beta) = f_1(\alpha) f_2(\beta)$$

and

$$N = \frac{\sigma'}{2\sigma} f_1(0)\, \mu_2,$$

μ_2 being the first absolute moment of the variable β.

Expressing classically σ and σ' as a function of the power spectrum $\Phi(f)$ of f (t), we shall obtain the general formula :

$$N_0 = \frac{1}{2\pi^2 \mu_2 f_1(0)} \left[\frac{\int_0^{+\infty} f^2 \Phi(f)\, df}{\int_0^{+\infty} \Phi(f)\, df} \right]^{\frac{1}{2}} \qquad (2)$$

which restores, as a particular case, the Rice formula relative to Gaussian processes :

$$N_0^2 = \frac{\int_0^{+\infty} f^2 \Phi(f)\, df}{\int_0^{+\infty} \Phi(f)\, df}. \qquad (3)$$

2. - The Rice formulas and their generalization allow to define a practical method for obtaining spectral densities. In order to alleviate the account the demonstration will be made for Gaussian processes, that is, after formula (3).

While the Rice formulas usually serve to forecast the mean number of zeros starting from the knowledge of the power spectrum, we shall strive to determine the power spectrum starting from the zeros of the function.

Assuming that the function f (t) is sent into a low-pass filter characterized by a cutoff frequency f, the square $N_0^2(f)$ of the number of zeros will be function of the frequency f :

$$N_0^2(f) = \frac{\int_0^f \alpha^2 \Phi(\alpha)\, d\alpha}{\int_0^f \Phi(\alpha)\, d\alpha}. \qquad (4)$$

Postulating, for the sake of simplification,

$$\Phi_1(f) = \int_0^f \Phi(\alpha)\, d\alpha,$$

we obtain by deriving formula (4) :

$$\frac{\partial N_0^2}{\partial f} = \frac{f^2 \Phi(f)\, \Phi_1(f) - \Phi(f) \int_0^f \alpha^2 \Phi(\alpha)\, d\alpha}{\Phi_1^2(f)},$$

that is, making use of the relation (4) :

$$\frac{\partial N_0^2}{\partial f} = \frac{[f^2 - N_0^2]\, \Phi(f)}{\Phi_1(f)}. \qquad (5)$$

By inversion of this formula, we obtain :

$$\Phi(f) = \frac{\partial N_0^2}{\partial f} \frac{\Phi_1(f)}{f^2 - N_0^2}.$$

The process for obtaining the power spectrum is as follows: we compute the cutoff frequency f with initial value f_o and we measure $N_0^2(f_0)$. We then bring the cutoff frequency to the value

$$f_1 = f_0 + \delta f.$$

We measure $N_0^2(f_1)$ and we postulate arbitrarily $\Phi_1(f_1) = 1$.

From the measurements of N_0^2 at f_0 and f_1 we derive $\partial N_0^2/\partial f$ and

$$\Phi(f_1) = \frac{\partial N_0^2}{\partial f} \frac{1}{f_1^2 - N_0^2}.$$

In order to carry out the further step, corresponding to

$$f_2 = f_1 + \delta f,$$

we measure $N_0^2(t_2)$, and we derive therefrom $\partial N_0^2/\partial f$, and since

$$\Phi_1(f_2) = 1 + \Phi(f_1)\,\delta f,$$

we have

$$\Phi(f_2) = \frac{\partial N_0^2}{\partial f} \frac{1 + \Phi(f_1)\,\delta f}{f_2^2 - N_0^2}.$$

Generally speaking, to the frequency

$$f_n = f_0 + n\,\delta f$$

corresponds a power spectrum

$$\Phi(f_n) = \frac{\partial N_0^2}{\partial f} \frac{\Phi_1(f_{n-1}) + \Phi(f_{n-1})\,\delta f}{f_n^2 - N_0^2}.$$

This method, which allows a practical iteration of the spectral densities provides this function only with the approximation to one factor. This uncertainty is eliminated by measuring effectively $\Phi_1(f_N)$ for a cutoff frequency f_N. $\Phi_1(f_N)$ is no other than the typical gap corresponding to the function $f(t)$, smoothed to frequency f_N.

3. - The advantages of this method are the following:

a. - The use of very selective filters, of which the calibrations, generally delicate and often unreliable, are avoided.

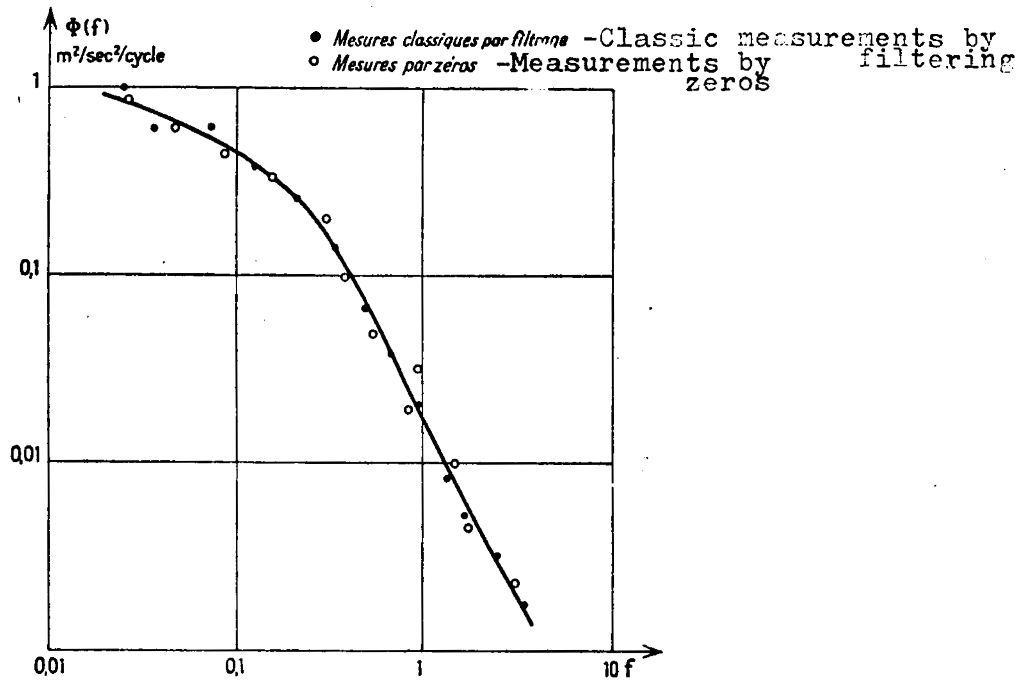
b.- Any defect of the signal studied is of limited influence and does not produce the disastrous effect of a transient response of the selective filter.

c.- the only measurement requiring calibration is that of $\Phi_1(f_N)$ for the "framing" of the spectral density.

d.- the Rice formulas thus applied assure a practical "prewhitening" without which the measurement of spectra evolving very rapidly is vitiated by heavy errors.

e.- the only measurements carried out are those of one number of zeros. Made with pulse counters, they are vitiated with a relative error

of the order of $1/N$, N being the result of the count. (When processing turbulence, this error is less than 1 / 100).



The above figure shows a comparison of spectral densities relative to atmospheric turbulence obtained by the classical method of filtering with the zero method. It may be seen that the scattering of the points obtained is of same order (it is distinctly better, per second, at low frequencies).

****** THE END ******

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